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Common Fixed Point Theorem in Generalized Fuzzy Metric Spaces for 6 Mapping Taking Integral Type Mappings

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ABSTRACT: Our aim of this paper is to obtain a common fixed point theorem for six self mappings of generalized metric space.

KEY WORDS: Self mapping, complete metric spaces, t-norm, common fixed point theorem.

I. INTRODUCTION

Many attempts have been made for proposing non-additive models of uncertainty. Most radical attempt was initiated by Zadeh [9] in 1965 with the publication of his paper "Fuzzy Sets". The notion of fuzzy set is a turning point in the development of mathematics. Consequently the last three decades were very productive of mathematics. Fuzzy fixed point theory has become an area of interest for specialists in fixed point theory.

In this paper we establish a general common fixed point theorems, which generalize the result of Singh and Chouhan [3,4], Singh, B. and Sharma [5,6,7], Som, T and Mukherjee Ray [8].

In 2002, Branciari [1] obtained a fixed point theorem for a single mapping satisfying an analogue of a Banach contraction principle for integral type inequality. After the paper of Branciari, a lot of research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying various known properties.

II. THEOREM

Theorem 2.1: (Branciari) [1] Let (X, d) be a complete metric space, $c\epsilon(0,1)$ and let $f : X : X$ be a mapping such that for each $x, y\epsilon X$,

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{d(x,y)} \varphi(t)dt$$

Where $\varphi: [0,+\infty) \to [0,+\infty)$ is a Lesbesgue – integrable mapping which is summable on each compact subset of $[0,+\infty)$, nonnegative, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t) > 0$, then f has a unique fixed point $a \in X$ such that for each $x\epsilon X, \lim_{n\to\infty} f^n x = a$

Theorem 2.2: [2] Let (X, d) be a complete metric space and f: X →X such that

$$\int_0^{d(fx,fy)} u(t)dt \le \alpha \int_0^{d(x,fx)+d(y,fy)} u(t)dt + \beta \int_0^{d(x,y)} u(t)dt + \gamma \int_0^{\max\{d(x,fy),d(y,fy)\}} u(t)dt,$$

For each x, y ϵ X with non-negative reals α, β, γ such that $2\alpha + \beta + 2\gamma < 1$, Where $u: [0,+\infty) \to [0,+\infty)$ is a Lesbesgue – integrable mapping which is summable,non-negative and such that for each $\epsilon > 0$, $\int_0^{\varepsilon} u(t)dt > 0$. Then f has a unique fixed point in X.

There is a gap in the proof of Theorem 2.2 In fact, the authors [2]

Used the inequality $\int_0^{a+b} u(t) \leq \int_0^a u(t)dt + \int_0^b u(t)dt$ for 0 ≤a<b, which is not true in general. The aim of the paper is to present in the presence of this inequality an extension of Theorem 2.2 using altering distance functions.

On taking the concept of Branciari we establish common fixed point theorem in generalized for 6 mappings taking integral type mapping.

Definition 2.3: The 3-tuple (X, S, *) is said to be a S-Fuzzy Metric Space if X is anArbitrary Set, * is a continuous t-norm and S is a Fuzzy set on $X^3 \times (0,\infty)$. Satisfying the following conditions.

(i) S(x, y, z, t) > 0
(ii) S(x, y, z, t) = 1 if and only if
(iii) S(x, y, z, t) = S(y, z,x,t)S (z, y, x, t) (Symmetry)
(iv) $S(x,y,z,r+s+t) \geq S(x,y,w,r) * S(x,w,z,s) * S(w,y,z,t)$
(Trtrahedral inequality)
(v) $S(x,y,z,.):(0,\infty) \to [0,1]$ is continuous for all $x,y,z,w\epsilon X$ and $r,s,t > 0$

Geometrically $S(x,y,z,t)$ represents the Fuzzy Perimeter of the triangle whose vertices are the points x,y and z with respect to $t > 0$.

III. MAIN RESULT

Theorem 3.1: Let A, B, P, Q, E and F be self mappings of a complete fuzzy metric space (X, S, *) with continuous t-norm $*$ defined by $a * b = \min\{(a,b) : a,b\epsilon[0,1]\}$ satisfying the following conditions

$AB(x) \subset F(x), PQ(x) \subset E(x)$ for all $x,y \epsilon X, t > 0$ … ...(1.1)

Mappings E and F are continuous. …….(1.2)

{AB, E} and {PQ, F} are compatible pairs of mapping for all $x,y\epsilon X, t > 0, k\epsilon[0,1]$ ... (1.3)

$$\int_0^{S(ABx,PQy,kt)} \varphi(t)dt \geq \int_0^J \varphi(t)dt \qquad \text{... (1.4)}$$

Where $J = \min\{S(ABx,Ex,t), S(PQy,Fy,t), S(Ex,Fy,t), \frac{S(ABx,Ex,2t)S(PQy,Fy,2t)}{S(Ex,Fy,t)}, \frac{S(ABx,Ex,2t)}{S(Ex,Fy,t)}, \frac{S(ABx,Fy,t)+S(PQy,Ex,t)}{2}\}$

For all $x,y\epsilon X, \lim S(x,y,t) \to 1 \ as\ t \to \infty$ ...(1.5)

Then AB, PQ, E and F have a unique Common fixed point in X.

Proof: Let $x_0 \epsilon X$ be arbitrary. Since AB(x)▫F(x). We can find a point x_1 in X such that $ABx_0 = Fx_1$. Also since PQ(x)▫E(x). We can choose a point x_2 with $PQx_1 = Ex_2$.

Using this argument repeatedly we can construct sequence $\{y_n\}$ in X such that $y_{2n-1} = Fx_{2n-1} = ABx_{2n-2}$ and

$$y_{2n} = Ex_{2n} = MNx_{2n-1}, n = 1,2,3, \dots \dots \dots$$

From (1.4) we have

$$\int_0^{S(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt = \int_0^{S(ABx_{2n},PQx_{2n+1},kt)} \varphi(t)dt$$

$$\geq \int_0^{J_1} \varphi(t)dt$$

Where $J_1 = \min\{S(ABx_{2n},Ex_{2n},t), S(PQx_{2n+1},Fx_{2n+1},t) S(Ex_{2n},Fx_{2n+1},t), \frac{S(ABx_{2n},Ex_{2n},2t)S(PQx_{2n+1},Fx_{2n+1},2t)}{S(Ex_{2n},Fx_{2n+1},t)}, \frac{S(ABx_{2n},Ex_{2n},2t)}{S(Ex_{2n},Fx_{2n+1},t)}, \frac{S(ABx_{2n},Fx_{2n+1},t)+S(PQx_{2n+1},Ex_{2n},t)}{2}\}$

$$\geq \int_0^{J_2} \varphi(t)dt$$

Where $J_2 = \min\{S(y_{2n},y_{2n+1},t), S(y_{2n+2},y_{2n+1},t), S(y_{2n},y_{2n+1},t), \frac{S(y_{2n+1},y_{2n},2t)S(y_{2n+2},y_{2n+1},2t)}{S(y_{2n},y_{2n+1},t)}, \frac{S(y_{2n+1},y_{2n},2t)}{S(y_{2n},y_{2n+1},t)}, \frac{S(y_{2n+1},y_{2n+1},t)+S(y_{2n+2},y_{2n},t)}{2}\}$

$$\int_0^{S(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt \geq \int_0^{S(y_{2n},y_{2n+1},t)} \varphi(t)dt$$

Which implies in general

$$\int_0^{S(y_n,y_{n+1},kt)} \varphi(t)dt \geq \int_0^{S(y_{n-1},y_n,t)} \varphi(t)dt \qquad \text{.... (1.6)}$$

To prove that $\{y_n\}$ is a Cauchy Sequence we shall prove

$$S(y_n, y_{n+m}, t) \geq 1 - \lambda \qquad \text{...(1.7)}$$

is true for all $n \geq n_0$ and every $m \in N$

Here we use induction method from (1.6) we have

$$\int_0^{S(y_n,y_{n+1},t)} \varphi(t)dt \geq \int_0^{S(y_{n-1},y_n,\frac{t}{k})} \varphi(t)dt$$

$$\geq \int_0^{S(y_{n-2},y_{n-1},t/k^2)\dots\dots\dots S(y_0,y_1,t/k^n)} \varphi(t)dt \quad \to 1 \text{ as } n \to \infty$$

i.e. For $t > 0, \lambda \epsilon (0,1)$ we can choose $n_0 \epsilon N$ such that $S(y_n, y_{n+1}, t) \geq 1 - \lambda$

Thus (1.7) is true for m=1 suppose (1.7) is true for m then we shall prove that it is also true for m+1.
Using the definition of fuzzy metric space by (1.6) and (1.7) we have

$$\int_0^{S(y_n,y_{n+m+1},t)} \varphi(t)dt \geq \int_0^{\min\{S(y_n,y_{n+m},t/2), S(y_{n+m},y_{n+m+1},t/2)\}} \varphi(t)dt$$

$$\geq 1 - \lambda$$

Hence (1.7) is true for m+1. Thus $\{y_n\}$ is a Cauchy Sequence. By completeness for $(X, S, *)$, $\{y_n\}$ converges to some point z in X. Thus $\{ABx_{2n}\}, \{Ex_{2n}\}, \{PQx_{2n-1}\}$ and $\{Fx_{2n-1}\}$ also converges to z. Now $ABx_{2n} \to z$ and E is continuous . Hence $EABx_{2n} \to Ez$.

Thus for $t > 0, \lambda \epsilon (0,1)$ there exist an $n_0 \epsilon N$ such that

$S(EABx_{2n}, Ez, t/2) \geq 1 - \lambda$ for all $n > n_0$

Using (1.3) we have

$$\int_0^{S(ABEx_{2n},EABx_{2n},t/2)} \varphi(t)dt \to 1$$

$$\int_0^{S(ABEx_{2n},Ez,t/2)} \varphi(t)dt \geq \int_0^{\min\{S(ABEx_{2n},EABx_{2n},t/2),S(EABx_{2n},Ez,t/2)\}} \varphi(t)dt$$

$$\geq 1-\lambda$$

Hence $ABEx_{2n} \to Ez$ …..(1.8)

Similarly $PQFx_{2n-1} \to Fz$ ……(1.9)

Using (1.4) we have

$$\int_0^{S(ABEx_{2n},PQFx_{2n-1},kt)} \varphi(t)dt \geq \int_0^{J_3} \varphi(t)dt$$

$$\min\{S(ABEx_{2n},E^2x_{2n},t),S(PQFx_{2n-1},F^2x_{2n-1,t}),$$

$$S(E^2x_{2n},F^2x_{2n-1},t),$$

Where $J_{3=}\ \frac{S(ABEx_{2n},E^2x_{2n},2t)S(PQFx_{2n-1},F^2x_{2n-1},2t)}{S(E^2x_{2n},F^2x_{2n-1},t)},\frac{S(ABEx_{2n},E^2x_{2n},2t)}{S(E^2x_{2n},F^2x_{2n-1},t)},$

$$\frac{S(ABEx_{2n},F^2x_{2n-1},t)+S(PQFx_{2n-1},E^2x_{2n},t)}{2}\}$$

Taking limit as $n \to \infty$ and using (1.8)and (1.9) we get

$$\int_0^{S(Ez,Fz,kt)} \varphi(t)dt \geq \int_0^{S(Ez,Fz,t)} \varphi(t)dt$$

Which implies that

Ez = Fz ……(1.10)

Now

$$\int_0^{S(ABz,PQFx_{2n-1},kt)} \varphi(t)dt \geq \int_0^{J_4} \varphi(t)dt$$

$$\min\{\mathrm{S(ABz,Ez,t)},\mathrm{S}(\mathrm{PQFx}_{2n-1},\mathrm{F}^2\mathrm{x}_{2n-1,t}),$$

$$\mathrm{S(Ez,F^2x_{2n-1},t)},$$

Where $J_{4=}\ \frac{\mathrm{S(ABz,Ez,2t)S(PQFx_{2n-1},F^2x_{2n-1},2t)}}{\mathrm{S(Ez,F^2x_{2n-1},t)}},\frac{\mathrm{S(ABEz,Ez,2t)}}{\mathrm{S(Ez,F^2x_{2n-1},t)}},$

$$\frac{\mathrm{S(ABEz,F^2x_{2n-1},t)+S(PQFx_{2n-1},Ez,t)}}{2}\}$$

Taking the limit as $n \to \infty$ and using (1.9) and (1.10) we get

$$\int_0^{S(ABz,Fz,kt)} \varphi(t)dt \geq \int_0^{S(ABz,Fz,t)} \varphi(t)dt$$

Which implies that

ABz = Fz (1.11)

$$\int_0^{\mathrm{S(ABz,PQz,kt)}} \varphi(\mathrm{t})\mathrm{dt} \geq \int_0^{J_5} \varphi(\mathrm{t})\mathrm{dt}$$

Where $J_5 = \min\left\{S(ABz, Ez, t), S(PQz, Fz, t), S(Ez, Fz, t), \frac{S(ABz,Ez,2t)S(PQz,Fz,2t)}{S(Ez,Fz,t)}, \frac{S(ABEz,Ez,2t)}{S(Ez,Fz,t)}, \frac{S(ABEz,Fz,t)+S(PQz,Ez,t)}{2}\right\}$

$$\int_0^{S(ABz,PQz,kt)} \varphi(t)dt \geq \int_0^{J_6} \varphi(t)dt$$

Where $J_6 = \min\left\{S(Fz, Ez, t), S(PQz, ABz, t), S(Ez, Fz, t), \frac{S(ABz,Ez,2t)S(PQz,Fz,2t)}{S(Ez,Fz,t)}, \frac{S(ABEz,Ez,2t)}{S(Ez,Fz,t)}, \frac{S(ABEz,Fz,t)+S(PQz,Ez,t)}{2}\right\}$

Which implies that

ABz = MNz (1.12)

Using (1.10) and (1.12) we get

ABz = MNz = Ez = Fz (1.13)

Now $\int_0^{S(ABx_{2n},PQz,kt)} \varphi(t)dt \geq \int_0^{J_7} \varphi(t)dt$

Where $J_7 = \min\left\{S(ABx_{2n}, Ex_{2n}, t), S(PQz, Fz, t), S(Ex_{2n}, Fz, t), \frac{S(ABx_{2n}x_{2n},2t)S(PQz,Fz,2t)}{S(Ex_{2n},Fz,t)}, \frac{S(ABx_{2n},Ex_{2n},2t)}{S(Ex_{2n},Fz,t)}, \frac{S(ABx_{2n},Fz,t)+S(PQz,Ex_{2n},t)}{2}\right\}$

$$\geq \int_0^{J_8} \varphi(t)dt$$

Where $J_8 = \min\left\{S(Ex_{2n}, Fz, t), S(ABx_{2n}, Ex_{2n}, t), S(PQz, Fz, t), \frac{S(ABx_{2n},Ex_{2n},2t)S(PQz,Fz,2t)}{S(Ex_{2n},Fz,t)}, \frac{S(ABx_{2n},Ex_{2n},2t)}{S(Ex_{2n},Fz,t)}, \frac{S(ABx_{2n},Fz,t)+S(PQz,Ex_{2n},t)}{2}\right\}$

Taking limit as $n \to \infty$ and using (1.13) we get

$$\int_0^{S(z,PQz,kt)} \varphi(t)dt \geq \int_0^{S(z,PQz,t)} \varphi(t)dt$$

Which implies that

Z = MNz (1.14)

Thus z is a common fixed point of AB, PQ, E and F. For the uniqueness let w be another common fixed point of said mappings. Then from (1.14)

$$\int_0^{S(ABz,PQw,kt)} \varphi(t)dt \geq \int_0^{J_9} \varphi(t)dt$$

Where $J_9 = \min\{S(ABz, Ez, t), S(PQw, Fw, t), S(Ez, Fw, t)$

$$\frac{S(ABz, Ez, 2t)S(PQw, Fw, 2t)}{S(Ez, Fw, t)}, \frac{S(ABz, Ez, 2t)}{S(Ez, Fw, t)}, \frac{S(ABz,, Fw, t) + S(PQw, Ez, t)}{2}$$

i.e. $\int_0^{(z,w,kt)} \varphi(t)dt \geq \int_0^{S(z,w,t)} \varphi(t)dt$

Hence , z = w.This completes the proof.

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